1. **C** We have that $2(x_1+2y_1)-(2x_1+3y_1)=y_1=2(5)-8=2$, so $x_1=5-2y_1=1$. Thus, $x_1y_1=2$.
2. **B** This is a line with slope -1.
3. **C** We solve to obtain $S=[0,2)$ and $T=\left(4/3,2\right]$. The intersection is $\left(4/3,2\right)$.
4. **C** We add to obtain $5x+5y=6$, so $x+y=6/5=1.2$.
5. **A** Note that this only occurs where $5k(3x_1+2y_1)-k(2x_1+3y_1)=k(13x_1+7y_1)=0$, so $a=13k$ and $b=7k$. Since we require *a* and *b* to be positive, and 13 and 7 are relative prime, we can only have $k=1,2,3,...,7$. Any larger *k* would cause *a* to exceed 100.
6. **C** We can factor the inequality as $(x-5)(x+1)(x+2)(x+3)>0$, so we need either all four, exactly 2, or none of these factors to be positive. Since $x-5<x+1<x+2<x+3$, all four will be positive if $x-5>0$, or $x>5$. Exactly two will be positive if $x+2>0$ and $x+1<0$, which gives $-2<x<-1$. Finally, all four will be negative if $x+3<0$, or $x<-3$. Thus, the solution set is $x<-3$ or $-2<x<-1$ or $x>5$.
7. **B** Checking the intervals between the zeroes, we find the solutions as $n=6,7,...,22$ and $n=69,70,...,99$ for a total of $17+31=48$.
8. **C** Set $x^2-x-1=1$ which gives $x=2,-1$. Set $x^2-x-1=-1$ which gives which gives $x=0,1$. We need to check these to make sure the exponent is even since the base is negative: $x=1$ is not a valid solution since $(-1)^{1+2}=-1$. Finally, set $x+2=0$ to obtain $x=-2$. The only distinct integer solutions are $\{-2,-1,0,2\}$ for a total of 4.
9. **C** Each term in the series is positive, so the sum increases for each value of *n*. The largest possible value of *a* is $\sum_{k=0}^{1}\frac{k}{3^k}=\frac{1}{3}$ and the smallest possible value of *b* is $\sum_{k=0}^{\infty}\frac{k}{3^k}=S=\frac{1}{3}+\frac{2}{9}+\frac{3}{27}+...$ Then, $3S=1+\frac{2}{3}+\frac{3}{9}+...$ so $2S=1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...=\frac{1}{1-1/3}=3/2$, so the smallest possible value of b is $S=3/4$. The desired sum is $1/3+3/4=4/12+9/12=13/12$.
10. **C** Let $a=3^x$; then $a^2-30a+100=0$, so $a=\frac{30\pm\sqrt{900-400}}{2}=\frac{30\pm10\sqrt{5}}{2}=15\pm5\sqrt{5}$. Both of these are positive, so they are both valid solutions. Then, $x=\log_3(a)$ so the sum of the solutions is $\log_3(15+5\sqrt{5})+\log_3(15-5\sqrt{5})=\log_3(100)=\frac{\log(100)}{\log(3)}=\frac{2}{\log(3)}$.
11. **C** Distinct real roots occur when the discriminant is positive: $k^2-4>0$. Thus, we will have the desired condition for $k=3,4,5,6$. The probability is then $4/6=2/3$.
12. **C** This sum is $7\cdot5\cdot3\cdot1+8\cdot6\cdot4\cdot2=105+384=489$.
13. **B** We have $\log_2\left(\frac{2012!!}{1006!}\right)=\log_2\left(\frac{2012\cdot2010\cdot2008\cdots2}{1006\cdot1005\cdot1004\cdots1}\right)=\log_2\left(2^{1006}\right)=1006$.
14. **D** This is equivalent to $\sqrt{x^2+20x+100}>\sqrt{4x^2+4x+1}$, so we need $3x^2-16x-99<0$. The roots of $3x^2-16x-99=(3x+11)(x-9)$ are $-11/3$ and 9, so the valid integer solutions for *x* are $-3,-2,\ldots,8$ for a total of 12 solutions.
15. **A** This is $(-2012)^2-4(2012)(2012)=2012^2-4\cdot2012^2=-3\cdot2012^2$.

16. **C** Drawing a graph, this is a square with vertices at $(\pm C, 0)$ and $(0, \pm C)$. It has side length $\sqrt{C^2 + C^2} = C\sqrt{2}$. The area is then $\left(\sqrt{2}C\right)^2 = 2C^2$.

17. **D** Drawing a graph, this is a square with vertices at $(\pm C, \pm C)$ and $(\pm C, \mp C)$. It has side length $C + C = 2C$. The area is then $(2C)^2 = 4C^2$.

18. **B** Expanding gives $\frac{2x+2}{x^2+2x} = 1$, so $x^2 + 2x = 2x + 2 \Rightarrow x^2 = 2$. Then, $x^6 = \left(x^2\right)^3 = 8$.

19. **A** Using the triangle inequality, the third side length k can be $k = 2, 3, \ldots, 8$. The perimeter of each triangle is $9 + k$, so we desired $\sum_{k=2}^{8}(9+k) = 9 \cdot 7 + \frac{8 \cdot 9}{2} - 1 = 63 + 36 - 1 = 98$.

20. **C** Following the hint, we write $(\alpha-\beta)(\alpha+\beta)+(\gamma-\delta)(\gamma+\delta) = 2012$, so $(\alpha-\beta)(\alpha+\beta)+(\gamma-\delta)(\gamma+\delta) = \alpha+\beta+\gamma+\delta \Rightarrow (a-\beta-1)(\alpha+\beta)+(\gamma-\delta-1)(\gamma+\delta) = 0$. Since these are positive integers with $\alpha > \beta > \gamma > \delta$, this is only possible if $\alpha - \beta = \gamma - \delta = 1$. Thus, $\alpha + (\alpha - 1) + \gamma + (\gamma - 1) = 2012$, so $\alpha + \gamma = 1007$. Since δ is a positive integer, it must be at least 1, so γ is at least 2. This gives the maximum possible value of α as $1007 - 2 = 1005$. Specifically, it occurs in the solution $(1005, 1004, 2, 1)$.

21. **A** This is $\begin{vmatrix} 2 & 1 & 1 \\ 7 & 3 & -2 \\ 3 & -6 & 10 \end{vmatrix} = 2\begin{vmatrix} 3 & -2 \\ -6 & 10 \end{vmatrix} - 1\begin{vmatrix} 7 & -2 \\ 3 & 10 \end{vmatrix} + 1\begin{vmatrix} 7 & 3 \\ 3 & -6 \end{vmatrix} = 36 - 76 - 51 = -91$.

22. **A** This is $\begin{vmatrix} 3 & 1 & 1 \\ 4 & 3 & -2 \\ 12 & -6 & 10 \end{vmatrix} = 3\begin{vmatrix} 3 & -2 \\ -6 & 10 \end{vmatrix} - 1\begin{vmatrix} 4 & -2 \\ 12 & 10 \end{vmatrix} + 1\begin{vmatrix} 4 & 3 \\ 12 & -6 \end{vmatrix} = 54 - 64 - 60 = -70$.

23. **B** There are $\binom{10}{3} = 120$ sets he could choose, but he needs to have 1 in the set (as long as 1 is there, it's surely the smallest). If he has a 1, there are 9 numbers to choose the other 2 from, so there are $\binom{9}{2} = 36$ sets in which 1 is the smallest number. The probability is $36/120 = 3/10$.

24. **C** There are $\binom{10}{k}$ sets he could choose. For 2 to be the smallest number in the set, the set must contain 2 and it cannot contain 1. If he has a 2, there are 8 numbers to choose the other $k-1$ from, so there are $\binom{8}{k-1}$ sets in which 2 is the smallest number. Thus, $P_k = \frac{\binom{8}{k-1}}{\binom{10}{k}}$

$$= \frac{\frac{8!}{(k-1)!(9-k)!}}{\frac{10!}{k!(10-k)!}} = \frac{k(10-k)}{90} = \frac{10k - k^2}{90}$$ which has its maximum value when $k = -\frac{10}{2(-1)} = 5$.

25. **A** Each of the roots of $f(x-1)$ is 1 greater than the corresponding root of $f(x)$, so the sum of the 100 roots of $f(x)$ is 100 less than the sum of the roots of $f(x-1)$. Since the sum of the roots of $f(x-1)$ is $-0/1=0$, the sum of the roots of $f(x)$ is $0-100=-100$.

26. **B** Since $2012=2^2\cdot 503$, $\varphi(2012)=2012\left(1-\frac{1}{2}\right)\left(1-\frac{1}{503}\right)=\frac{2012\cdot 502}{1006}=2\cdot 502=1004$.

27. **B** The area of the quarter-circle is $\frac{\pi r^2}{4}=\frac{\pi\cdot 10000}{4}=2500\pi$.

28. **B** The diagonal of the square must be equal to the radius of the quarter-circle. Thus, if we denote the side length of the square by s, we have $s\sqrt{2}=100 \Rightarrow 2s^2=10000 \Rightarrow s^2=5000$.

29. **A** This length is $\sqrt{100^2+s^2}=\sqrt{10000+5000}=\sqrt{15000}=10\sqrt{150}=50\sqrt{6}$.

30. **D** Write $x^2-4x+6=(x-2)^2+2$ and $2x^2-8x+10=2(x-2)^2+2$, so both parabolas have vertex at $(2,2)$. Thus, $f(x)$ must also have its vertex at $(2,2)$, so we write $f(x)=a(x-2)^2+2$. Then, $f(12)=100a+2=182$, so $a=1.8$. Thus, $f(17)=1.8\cdot 15^2+2=407$.